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THESIS

A PARAMETRIC ANALYSIS OF THREE MODELS FOR DIRECT DELIVERY BY A NAVAL SUPPLY CENTER TO A NAVAL AIR REWORK FACILITY

by

Mary Ellen/Davidson

March 1981

Thesis Advisor: Alan W. McMasters

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A Parametric Analysis of Three Models for Direct Delivery by a Naval Supply Center to a Naval Air Rework Facility

by

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ABSTRACT

This thesis provides a parametric analysis of three models for direct delivery by a Naval Supply Center (NSC) to a Naval Air Rework Facility (NARF). The models include both scheduled and unscheduled deliveries. Parameters which were studied included the ratio of delay cost to delivery cost and the probability of a repair part being demanded by a component undergoing repair. The decision variables were the time between deliveries for scheduled deliveries and the number of units of an item delivered for unscheduled deliveries. The impact on the decision variables of varying the parameters was the major focus of the analysis. The results of the analysis suggest that scheduled delivery is a good direct delivery strategy for an NSC to use in supporting a NARF. However, the analysis has shown that the expected total cost for all three alteratives is very close. Therefore, the final criterion for which alternative should be chosen is essentially ease of usage and implementation.

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I. INTRODUCTION

With the consolidation of wholesale supply support between Naval Supply Centers (NSC) at Oakland, San Diego, and Norfolk and their neighboring Naval Air Stations, the question of providing supply support for local Naval Air Rework Facilities (NARF) with no degradation of that support is of primary concern. One possible answer is to provide on-site inventories at the NARF. This has the advantages of quick response to customer needs, smaller transportation costs, and smaller customer delay costs, and the disadvantage of increased costs of maintaining a separate inventory. Another possibility is support of the NARF by direct delivery from the NSC with no on-site inventory. And of course, a combination of these two is another possibility.

The optimum solution to the problem of supplying support to the NARF is a trade-off among customer needs, transportation costs and delay costs. McMasters [Ref. 1] has developed three direct delivery models as a first step in determining the best way to support the NARF. The complexity of the expected total cost formulas for all three alternatives requires a parametric analysis to understand the impacts of the various parameters.

This thesis will present a summary of the three models,

a detailed parametric analysis of them, and a brief discussion of the models under a time constraint. A modification to one of the models is then introduced and finally, an attempt is made to determine which model is most beneficial to the NSC. Formulas for all models will be presented without derivations; however, complete derivations may be found in McMasters [Ref. 1.].

II. SUMMARY OF THE MODELS

This chapter summarizes a deterministic demand direct delivery model and three random demand models. The deterministic model and its derivation are presented to illustrate the reasoning behind the random demand models analyses presented in this thesis. Since the details of their derivations are presented in Reference 1, only the results are presented here.

A. DETERMINISTIC DEMAND

If a demand from a customer occurs once every time period with certainty, it is said to be deterministic demand. Let CT be the cost of one round trip from a supply center to the NARF. If a truck is dispatched every time a demand is received and processed, the cost to deliver each unit is CT. If, however, the truck waits until k units have been demanded and processed, the average delivery cost per unit is

$$CT/k.$$

If the truck waits until it is full, say n units, the delivery cost per unit is minimized at

$$CT/n.$$

However, while k units accumulate, the units already required but undelivered accumulate delay costs for the

NARF. If the truck waits for k units to be accumulated and the delay cost for one unit for one time period is CD, the total average delay cost can be shown to be

$$(k-1)CD / 2.$$

To confirm this formula, assume one unit is needed every t units of time. If the truck waits for k units to accumulate, it will not leave until $(k-1)t$. During this time the units ordered but not delivered have accumulated delay. Specifically, the first unit ordered at time $t=0$ has been delayed $(k-1)t$ time units, the second unit ordered at $t=1$ has been delayed $(k-2)t$ time units, and so on until only the kth unit ordered at $(k-1)t$ has no delay. The total waiting time in periods of length t, then is

$$(k-1) + (k-2) + (k-3) + \ldots + 1 + 0,$$

which can be written $k(k-1)/2$. When this is multiplied by the delay cost per period, the result is the total delay cost

$$k(k-1)CD/2.$$

The average delay cost per unit is obtained by dividing by the number of units, giving the desired formula

$$(k-1)CD/2.$$

By adding the average shipping cost and average delay costs, the total average cost is

$$C(k) = CT/k + (k-1)CD / 2. \quad (1)$$

Figure 1 presents the total cost versus the number of units k for the deterministic model. Although the curves are

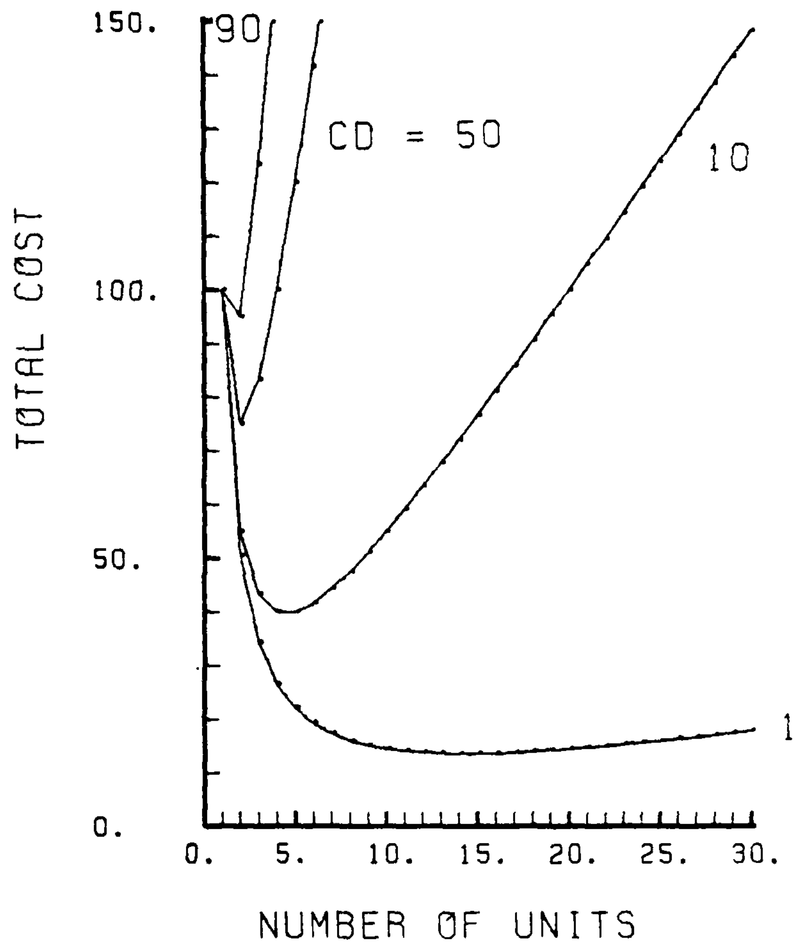


Figure 1. Total Cost Curves for the Deterministic Model with CT=100.

actually discrete, the points have been connected for clarity. With such discrete cost curves, the method of finite differences is often employed to find an optimal solution. Since it is desired to minimize the cost curve, optimum k is that k such that

$$C(k-1) > C(k) \leq C(k+1),$$

or equivalently , the largest k such that

$$C(k) - C(k-1) < 0 \quad \text{or}$$

the smallest k such that

$$C(k) - C(k+1) \leq 0.$$

Using equation (1), the second inequality above becomes

$$CT/k + CD(k-1)/2 - [\, CT/(k-1) + CD(k-2)/2 \,] < 0$$

which can easily be reduced to

$$k(k-1) < 2CT/CD \quad .$$

This final relationship allows a very simple computation to be made repeatedly until the k is found which satisfies the relationship. This method eliminates the requirement of evaluating equation (1) to search for optimum k. The only time equation (1) need be evaluated is after the optimal k is found and the total cost for that k is desired.

It is important to note that because one demand is known to occur every period, equation (1) is also the average cost per period. In the case of random demand, the expected cost per period is appropriate for making comparisons among the direct delivery strategies to be presented below.

The concept of a period is very important to the

following models. A period is defined as the time between inductions of components to be repaired at the NARF. For instance, if the NARF was scheduled to overhaul twenty engines of type A in one quarter, and if there were sixty working days in one quarter, the length of the period for the random models would be 3 days. Thus, the length of a period is determined by the work schedule at the NARF. It is assumed that the time spacing between demands for a repair part is equivalent to the time between inductions.

B. RANDOM DEMAND

If a repair part is not demanded every time period, but only in p percent of them, the total cost formula will differ from equation (1) and is dependent upon the delivery strategy. Three appear appropiate to consider for supply support of a NARF. They are:

1. The truck makes a delivery at the end of N periods of time if there has been at least one demand during the N periods.
2. The truck makes a delivery as soon as demands have accumulated to a specified number K.
3. The truck makes a delivery in the (N-1)st period following the first demand received after the last delivery.

These three alternatives respectively represent scheduled deliveries, unscheduled deliveries and a variant

of scheduled deliveries where the first demand marks the beginning of the time period before the next delivery. In each case, formulas have been derived for the expected average cost per period and the expected total delay [Ref. 1]. The purpose of the expected total delay formula is to allow for a time constraint to be imposed upon average expected delay. For comparison purposes, the expected number of units delivered under alternatives 1 and 3 and the number of periods between deliveries under alternative 2 have also been derived.

1. <u>Alternative 1</u>

The total expected average cost per period is

$$ECP(N) = \left[\frac{CT\,1-(1-p)^{N}}{N} + \frac{CD(N-1)p}{2}\right]\left[\frac{-\ln[1-(1-p)^{N}]}{(1-p)^{N}}\right] . \qquad (2)$$

The expected number delivered under alternative 1 is

$$E(X1) = Np \,/\, [1-(1-p)^{N}] \; . \qquad (3)$$

The average expected total delay is given by

$$ETD = (N-1) \,/\, 2 \qquad (4)$$

2. <u>Alternative 2</u>

The total average expected cost per period is

$$ECP(K) = CT\sum_{n=K}^{\infty} 1/n \binom{n-1}{K-1} p^{K} (1-p)^{n-K} + \frac{CD(K-1)}{2} \; . \qquad (5)$$

The expected number of periods between deliveries is

$$E(N2) = K/p. \qquad (6)$$

And the average expected total delay is

$$ETL = (K-1)/2p. \qquad (7)$$

3. Alternative 3

The total average expected cost is

$$TCP(N) = \left[CT + CI(N-1)\left[\frac{(N-2)p}{2} + 1\right]\right] \sum_{K=1}^{\infty} \frac{p(1-p)^{K-1}}{(K-1)+N}. \qquad (8)$$

As with alternative 2, bounds for equation (8) have been developed and will be discussed later. The expected number of units delivered is

$$E(KP) = 1 + (N-1)p. \qquad (9)$$

The average expected total delay is

$$ETD = (N-2)/2 + [1 - (1-p)^N] / p . \qquad (10)$$

C. COMPUTATIONAL APPROACH TO DETERMINING OPTIMAL VALUES

This section discusses the techniques used to determine optimal N and K using the expected cost formulas of the last section. While the method of finite defferences produced a simple relation to facilitate the determination of the optimal number of units or periods for the deterministic model, it was not as fruitful for the random demand models. The results of the finite differences method was at least as complicated as each of the expected cost formulas, so the expected costs equations (2), (5), and (8) were used in searching for the N or K value that minimized them.

Evaluation of equation (2), the total expected cost per period for alternative 1, presented no computational

problems. To determine the optimal number of units, N, for alternative 1, successive values of N beginning with N=1, were assumed and equation (2) was then evaluated. Using the concept of finite differences, the maximum N for which the total cost function continued to decrease was the optimum.

For alternative 2, the same approach was used in searching for optimal K. However, evaluation of the total cost formula, equation (5), is tedious because of the infinite sum. McMasters [Ref.1] conjectures that optimal K is the largest K such that

$$K(K-1) < 2pCT/CD$$

or it is one larger than that K. Although McMasters was unable to prove this conjecture, computational experience supports it. Using this inequality, two K values were determined and then used to evaluate equation (5). The K value with the minimum cost was the optimum. The shipping costs (CT) series in equation (5) was evaluated using an iterative method which was terminated when the new term contributed less than an additional .00001 of the previous CT sum.

In searching for optimal N for alternative 3, the same difficulty as with alternative 2 was created by the infinite sum in the total expected cost equation (8). McMasters [Ref. 1] also provides both an upper and lower bound for optimal N under alternative 3. The upper bound for N is the largest N such that

$$N(N-1) < 2/p\ [(CT/CD) - (1-p)]$$

The lower bound is the largest value of N which satisfies

$$(N-1)(N-2)p^2 + 2[(N-2)p + 1] < 2pCT/CD$$

The upper bound was chosen for computations because of its simpler form. So, an upper bound was calculated; then, successively smaller N values were used to evaluate equation (8), the total cost equation. Optimal N was the largest value of N for which the total expected cost continued to decrease.

III. PARAMETRIC ANALYSIS

This chapter will first present a discussion of the importance of the ratio of the shipping cost to the delay cost, and then examine the affect upon optimal solutions of varying certain parameters.

A. THE TRANSPORTATION-DELAY COST RATIO

Each of the expected total cost equations, (2), (3), and (5), consist of a sum of transportation cost and delay cost terms. The general form is

$$ECP = CT\ (A) + CD\ (B)$$

where A and B are specific terms applicable to each alternative and are functions of p and N or K. Minimizing an equation in this general form is equivalent to minimizing

$$\frac{ECP}{CT} = A + \frac{CD(B)}{CT} \qquad \text{or}$$

$$\frac{ECP}{CD} = \frac{CT(A)}{CD} + B \quad .$$

In either case, as long as the ratio of CT to CD are constant, even though they take on different values, the optimal solution will remain the same. While optimal N or K remain the same, the total cost changes by a multiplicative factor, i.e., if both CT and CD are doubled, the total cost will double.

Since the Public Works Center at Oakland has not been

able to provide an estimate of round-trip transportation costs from NSC, Oakland to NARF, Alameda, the analysis in this thesis will consider CT to be a constant value of $100. Delay costs per period must be provided by the NARF and are not yet available. Since delay costs are expected to vary by repair part and transportation costs are not, it was considered more meaningful to fix the CT value and vary the CD values.

B. GENERAL BEHAVIOR OF THE COST CURVES

Figure 2 provides graphs of the optimum expected total cost (ECP) versus the probability of a demand (p) for all three alternatives. The delay cost (CD) values were chosen merely to provide an indication of the behavior of the cost curves over a broad range of delay costs (CD) and are of no particular significance in themselves. As would be expected, an increase in either delay costs (CD) or probability of a demand (p) increases the optimum expected total cost (ECP). Note that at very small values of CD, the optimum expected total cost is extremely insensitive to changes in p. However, as CD increases a relatively small change in p causes significant increases in the total costs.

There are a few more interesting points to be gleaned from figure 2. First, within each alternative, the ECP curves for CD=30 and CD=50 are superimposed for small values of p. Investigation has revealed that they depart at that p

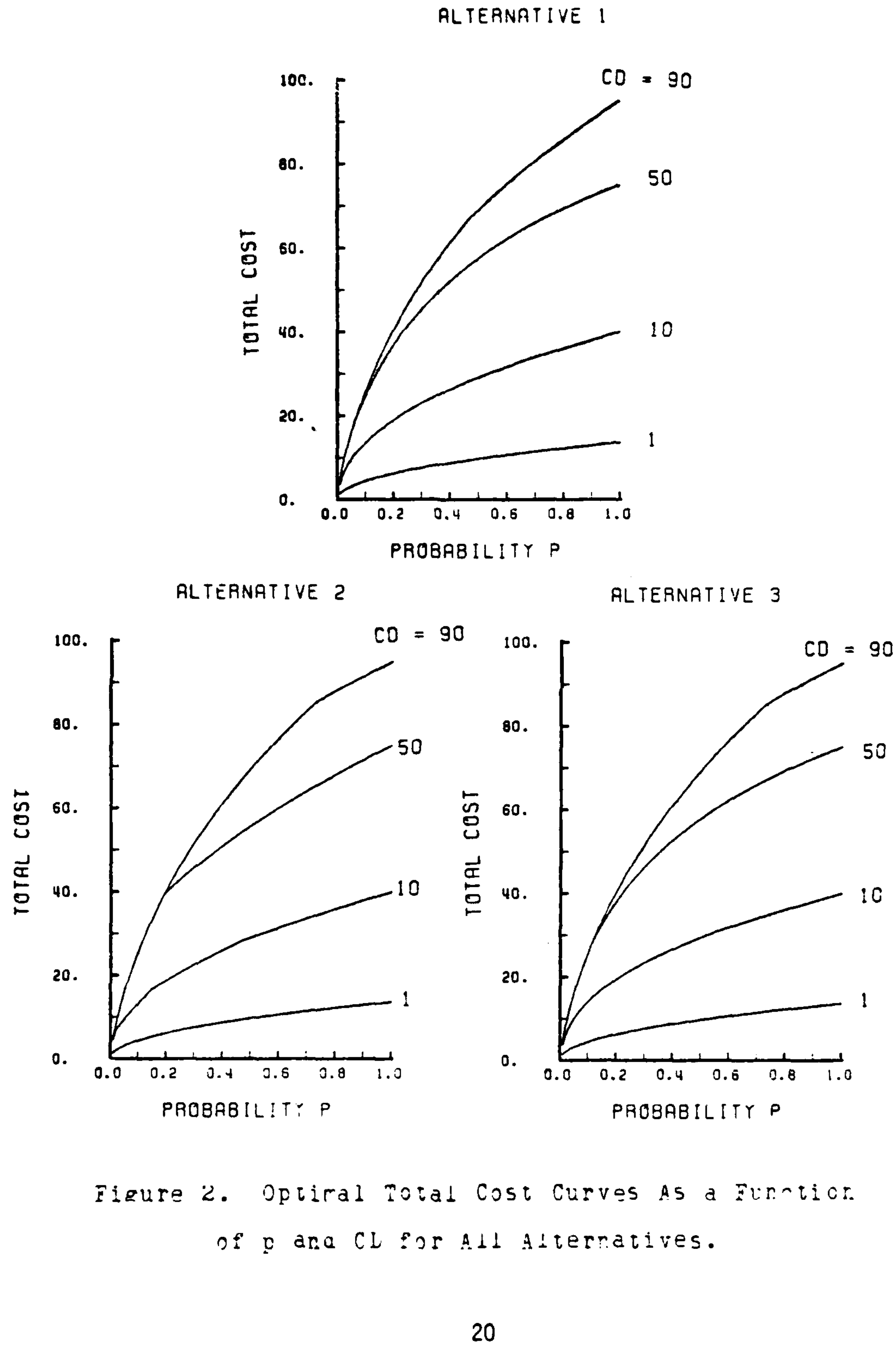


Figure 2. Optiral Total Cost Curves As a Function of p and CL for All Alternatives.

value that causes optimum N or K to change from 1 to 2 for CD=5M. In fact, the ECP curve for any CD value will share this same curve until the smaller CD value reaches the p value that causes optimum N or K to change from 1 to 2 provided the optimum N or K for very small p values is 1. The reason for this lies in the definition of the models. When N or K are 1, there are no delay costs, so no matter what the value of CD, only the transportation costs will contribute to the total expected cost. The other segments in the ECP curves are also correspond to constant values of N or K. In fact, the ECP curve may be thought of as a concatination of many different curves, one for each value of N or K. With large CD values these segments of the curve stand out because a value of N or K is optimal over a wide range of p values. As CD gets small so does the range over which a particular N or K is optimal. Thus, the curve for CD=1 appears to be smooth, lacking the segments of the larger CD value curves.

Another point worth stating is that the optimum total expected cost for a particular CD value is the same for all three alternatives when N or K = 1. Again this stems from the fact that the alternatives differ only in how expected delay costs are determined. If there are no delay costs, i.e., N or K =1, there is no diference in the alternatives. This is clearly shown in figure 3 by the graph for CD=100.

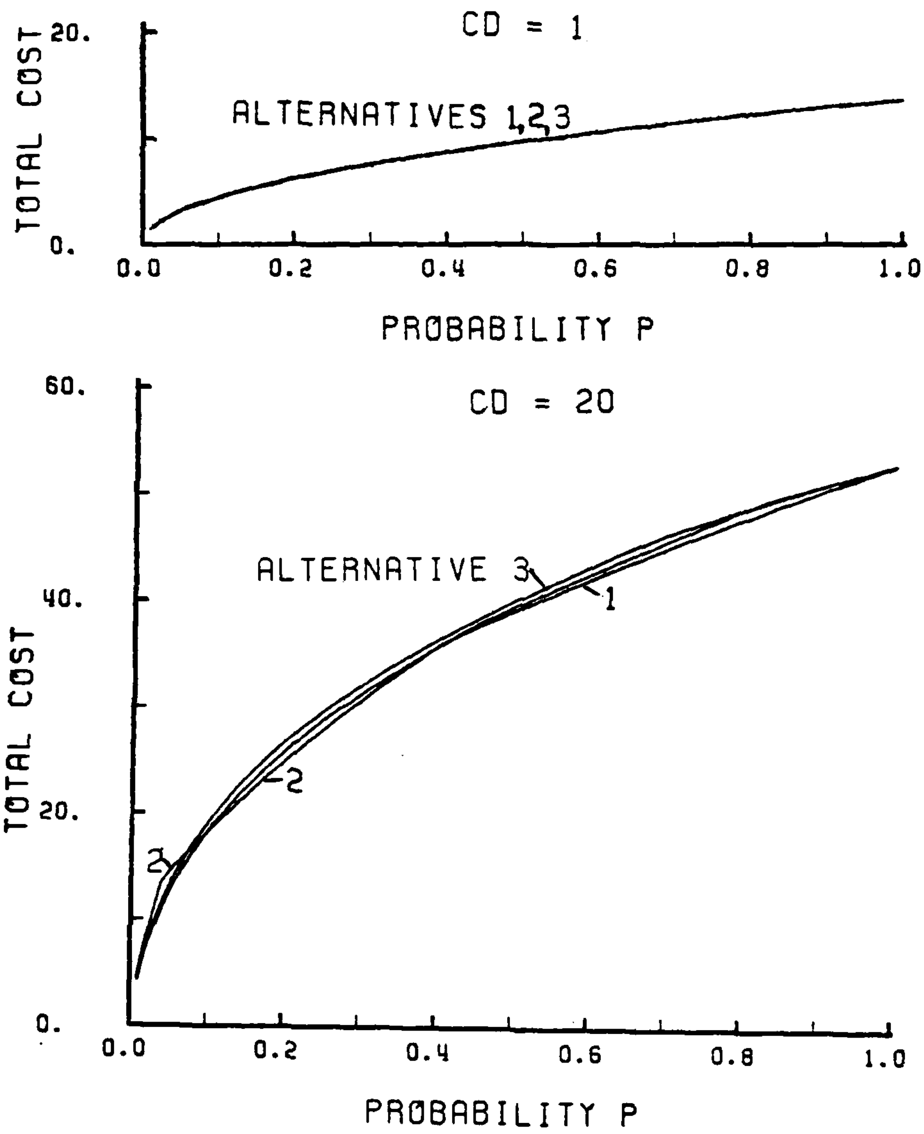


Figure 3. Optimal Total Cost Curves for Alternatives 1, 2, and 3 for a Given CD Value.

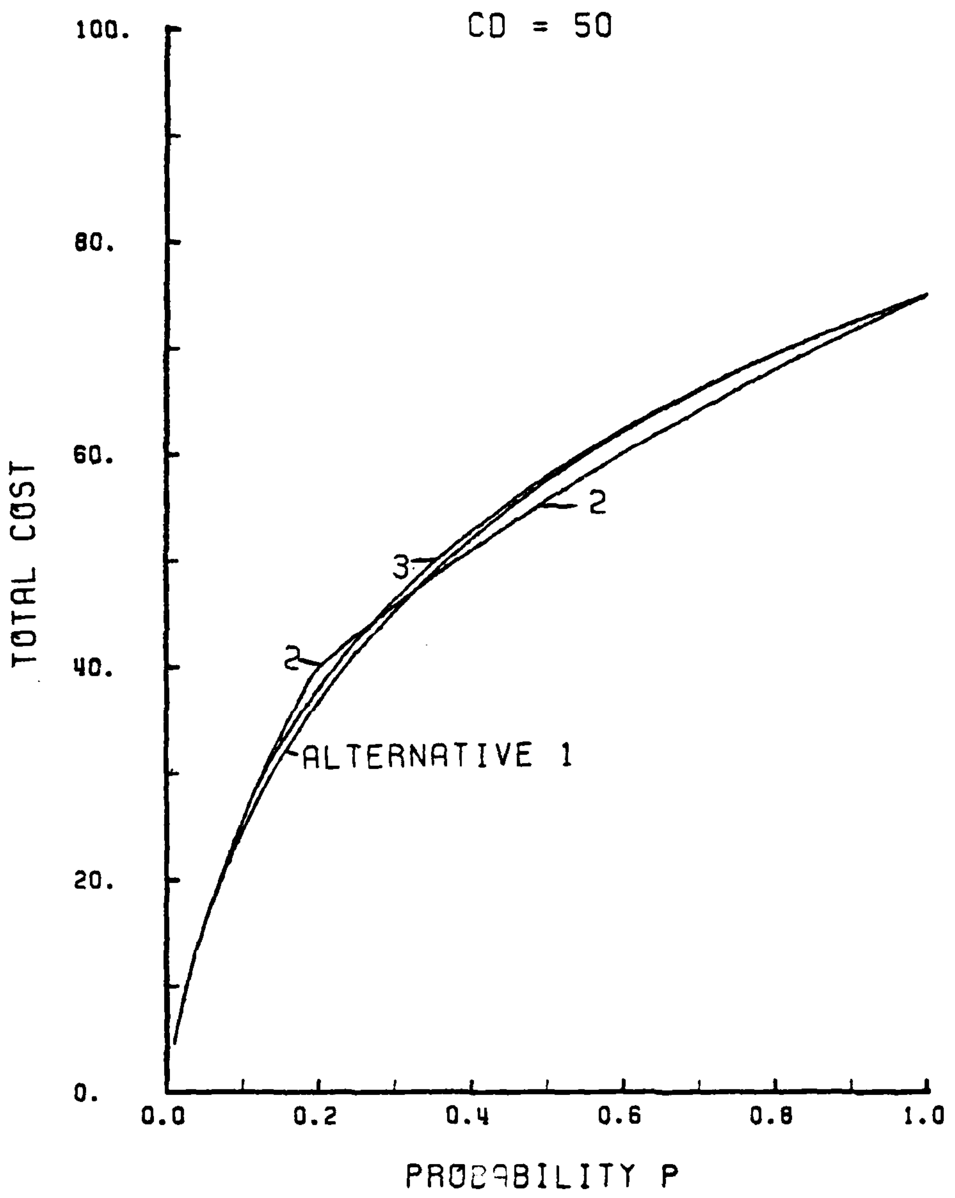
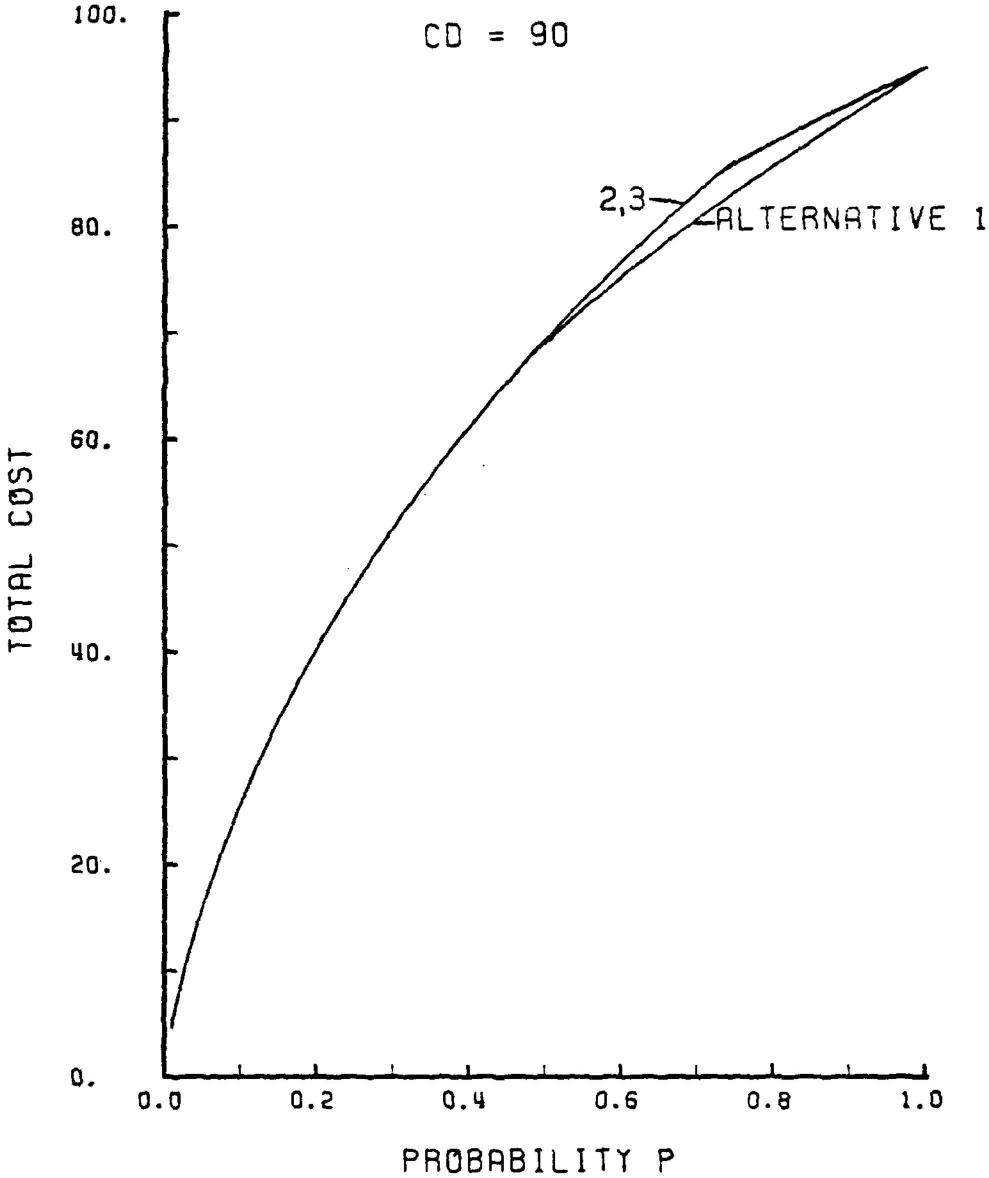


Figure 3. Continued.



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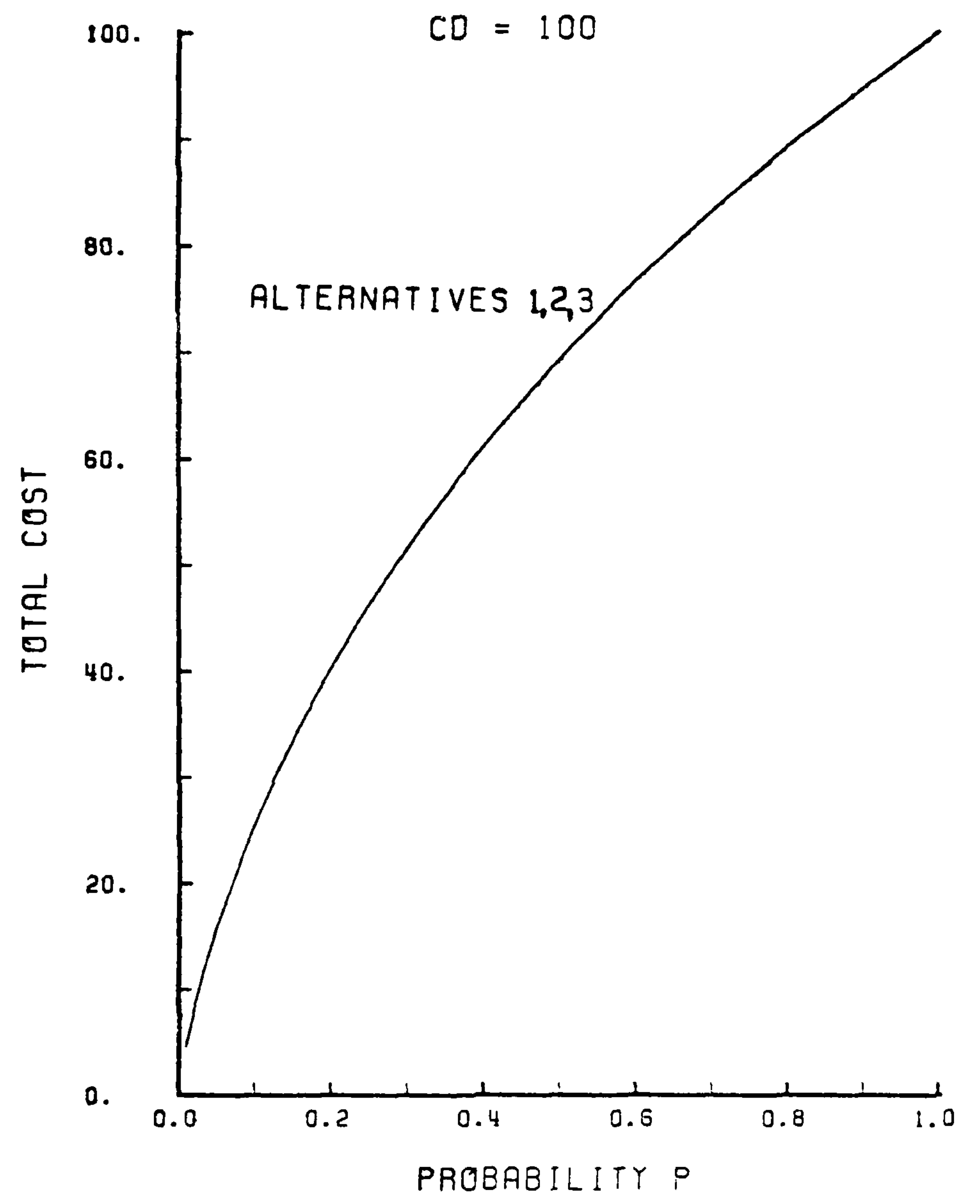

Figure 3. Continued.

It is also important to notice that there is little difference in the total expected optimum cost for any alternative. All three alternative cost curves have been plotted on the same graph for various CD values in figure 4. Through the whole spectrum of CD values from 1 to 100, there is little difference in total cost at any point. Differences are indistinguishable for CD=1, while they are perceptible though not substantial for CD = 20, 50, and 90. The CD=100 graph shows all three superimposed with no difference among any of the alternatives. This is because optimal N or K are unity and hence only the delivery cost term is positive in the expected total cost equations. For the three CD values that show differences among the alternatives, there is no one alternative that always provides the lowest total cost; rather, over the range of all p values, the most favorable alternative varies between alternatives 1 and 2. For the given CD values, it is interesting to note that alternative 3 never produces the lowest total cost.

Figures 4 through 6 depict average total cost versus N or K for three specific p values. Again, these curves are discrete, but the points have been connected for clarity. Since all three models reduce to the deterministic model at p=1.0, figure 1 (presented in Chapter II) reflects all alternatives for p=1.0. All three alternatives show the same trends. Once again the low CD values produce a very flat curve showing insensitivity to N or K values while the

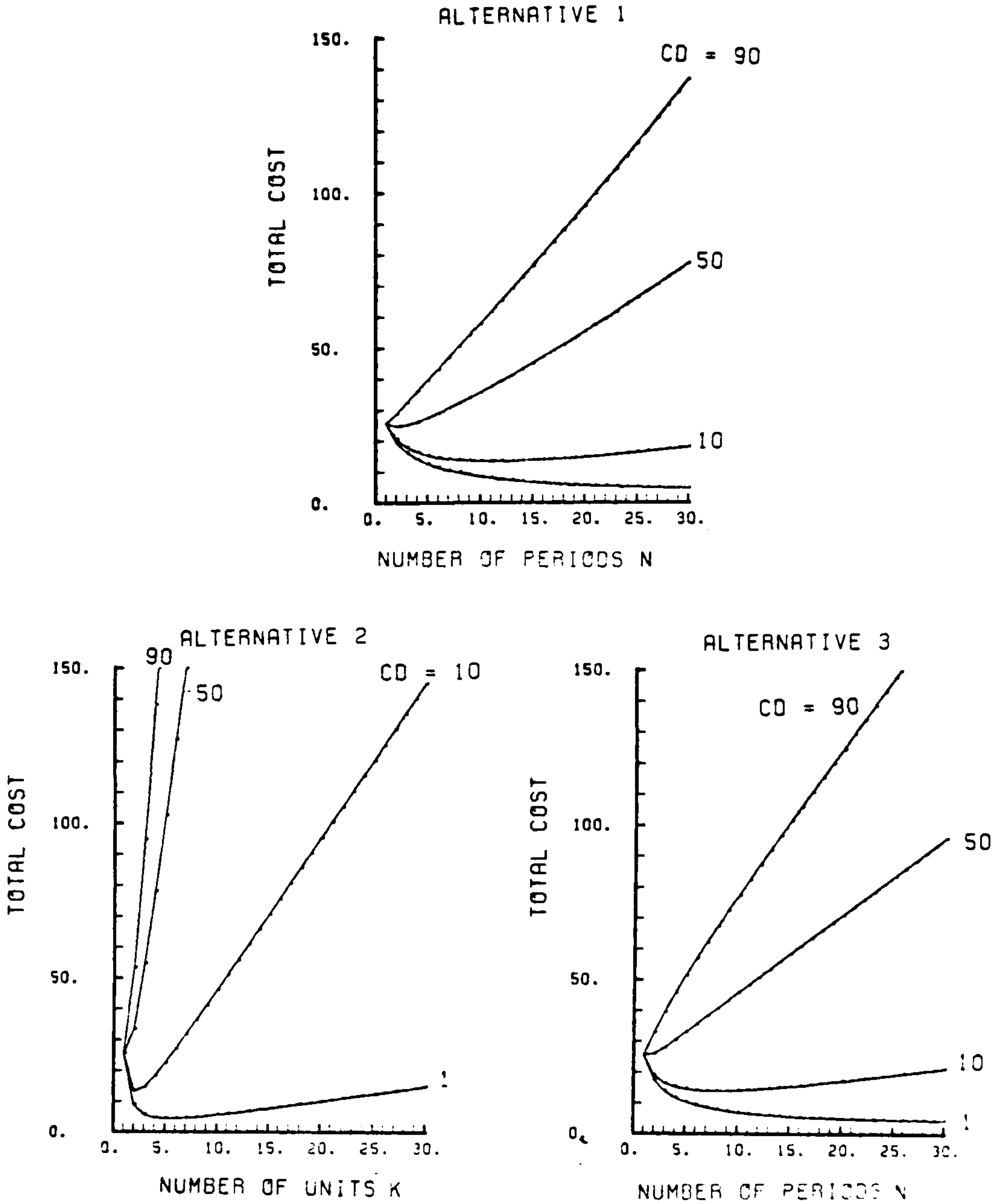

Figure 4. Total Cost Curves as a Function of CD and the Decision Variables for p=0.1.

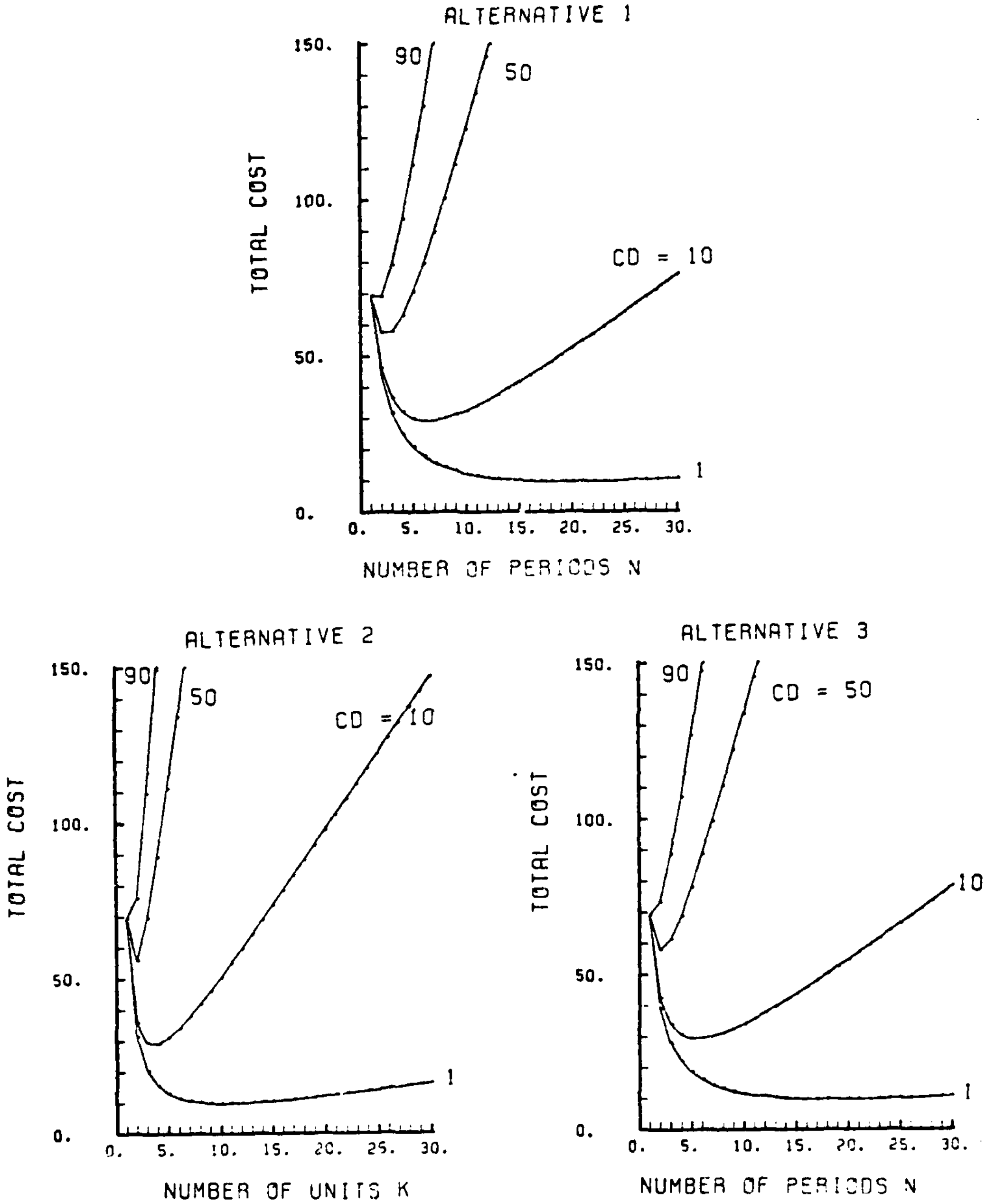

Figure 5. Total Cost Curves as a Function of CD and the Decision Variables for p=0.5.

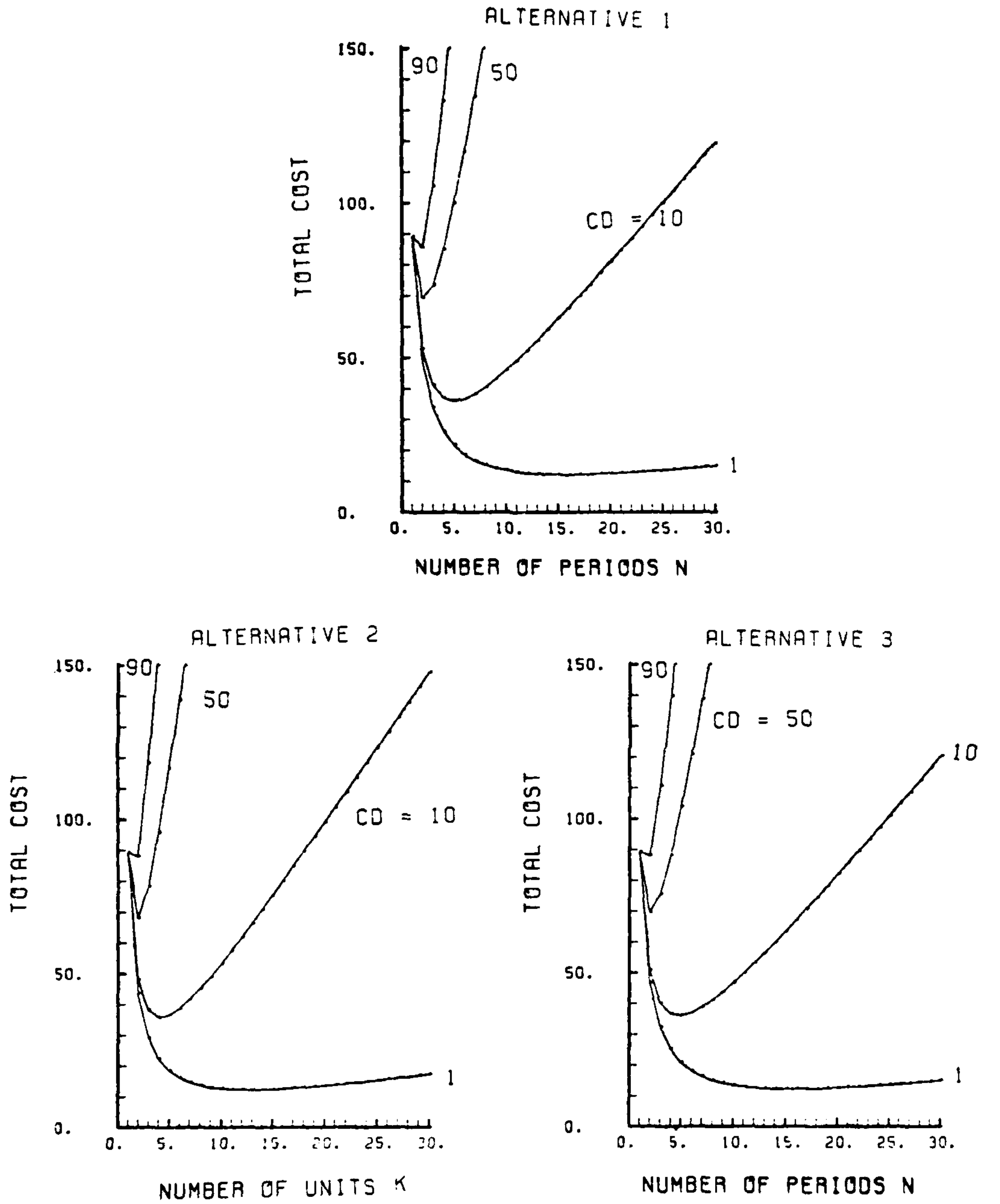

Figure 6. Total Cost Curves as a Function of CD and the Decision Variables for p=0.5.

higher the CD value, the more U-shaped the TC curve becomes. The same behavior also occurs as p increases.

For particular CD values, figures 7, 8, and 9 show the stair-step function of optimal N or K for alternatives 1, 2, and 3 respectively versus the probability of a demand. The behavior of alternative 2 is consistent for all values of CD given and indicates that as the probability of a demand increases, the optimal number of units to be accumulated before a delivery is made increases. However, alternatives 1 and 3 display behavior not consistent across the p values. All three given CD values for alternatives 1 and 3 show an increase in optimal N as p increases for very small values of p. In addition to this, the functions for CD = 20 and 50 also show a decrease in optimal N as p increases for large values of p. The key to this behavior lies in figure 10. For CD = 20, 50, and 90, the delivery cost term and the delay cost term have been plotted separately for several N values for alternative 1. For N=1, the only term involved in the total cost is the delivery term. However, for N>1 the delivery cost term quickly flattens out as p increases. On the other hand, the delay cost term increases approximately linearly with p. For small p values, the savings in the delivery cost term realized by an increase in N is more than the delay cost increase. However, when p becomes large the savings in delivery costs are overwhelmed by the increases in delay costs and lower N values again become optimal.

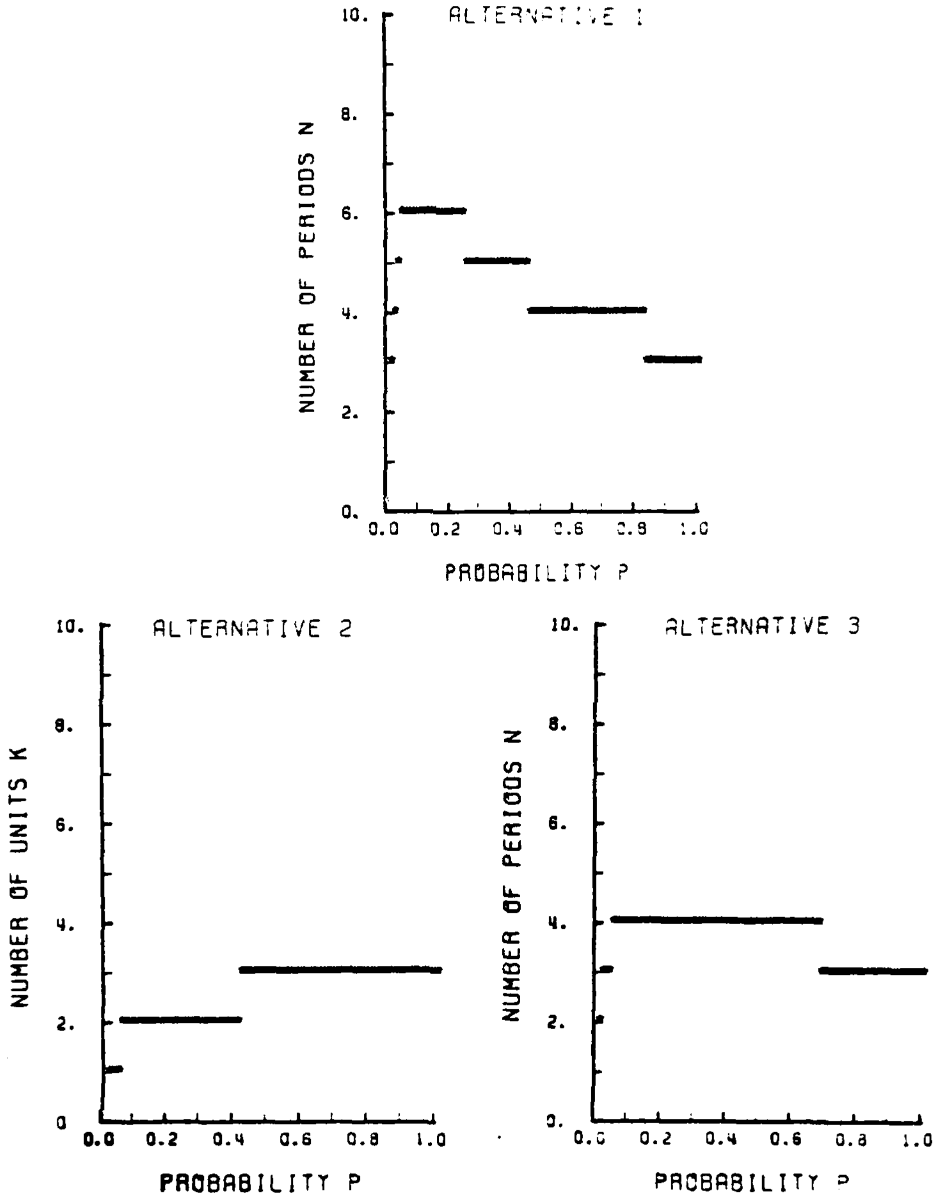

Figure 7. Optimal N or K for CI=20.

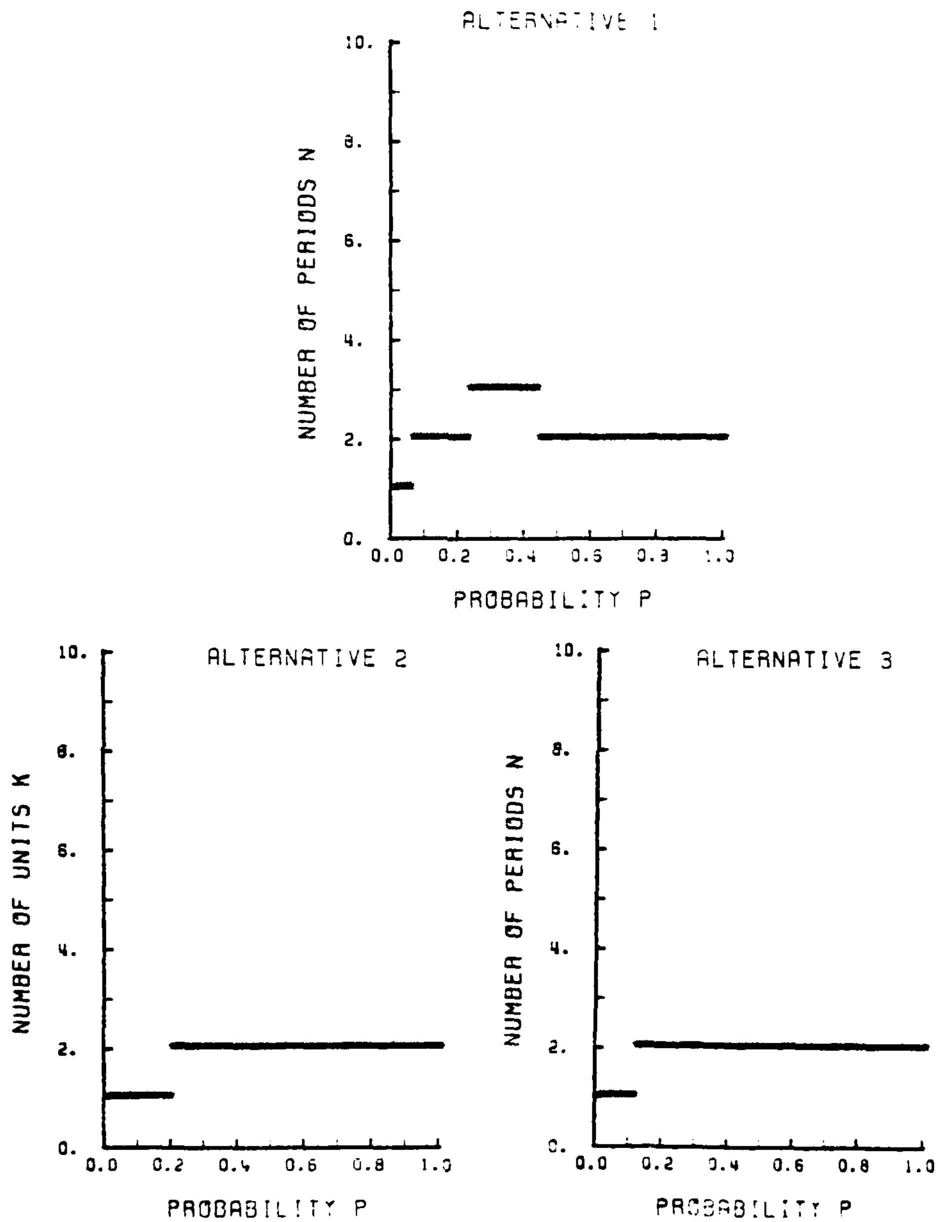

Figure 8. Optimal N or K for CD=50.

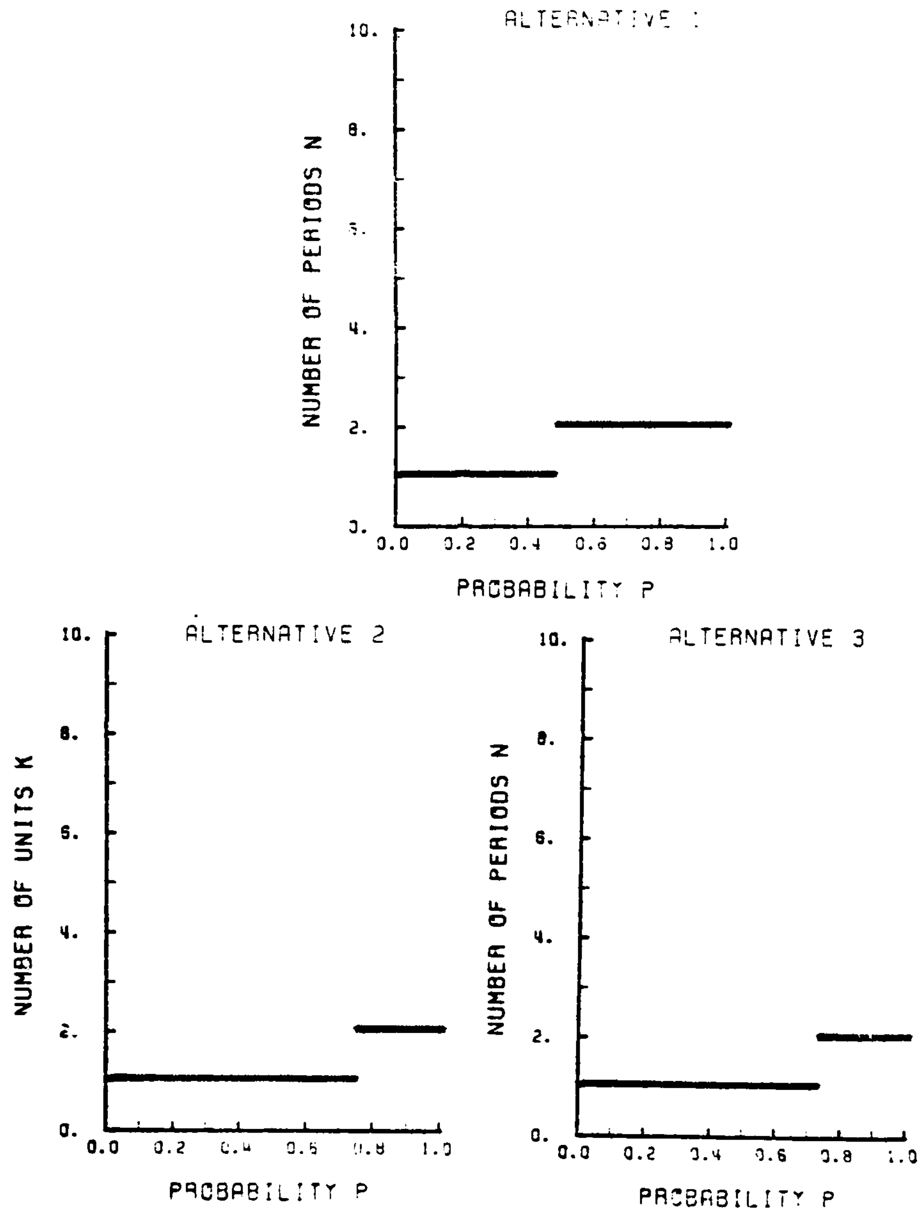

Figure 9. Optimal N or K for CD=90.

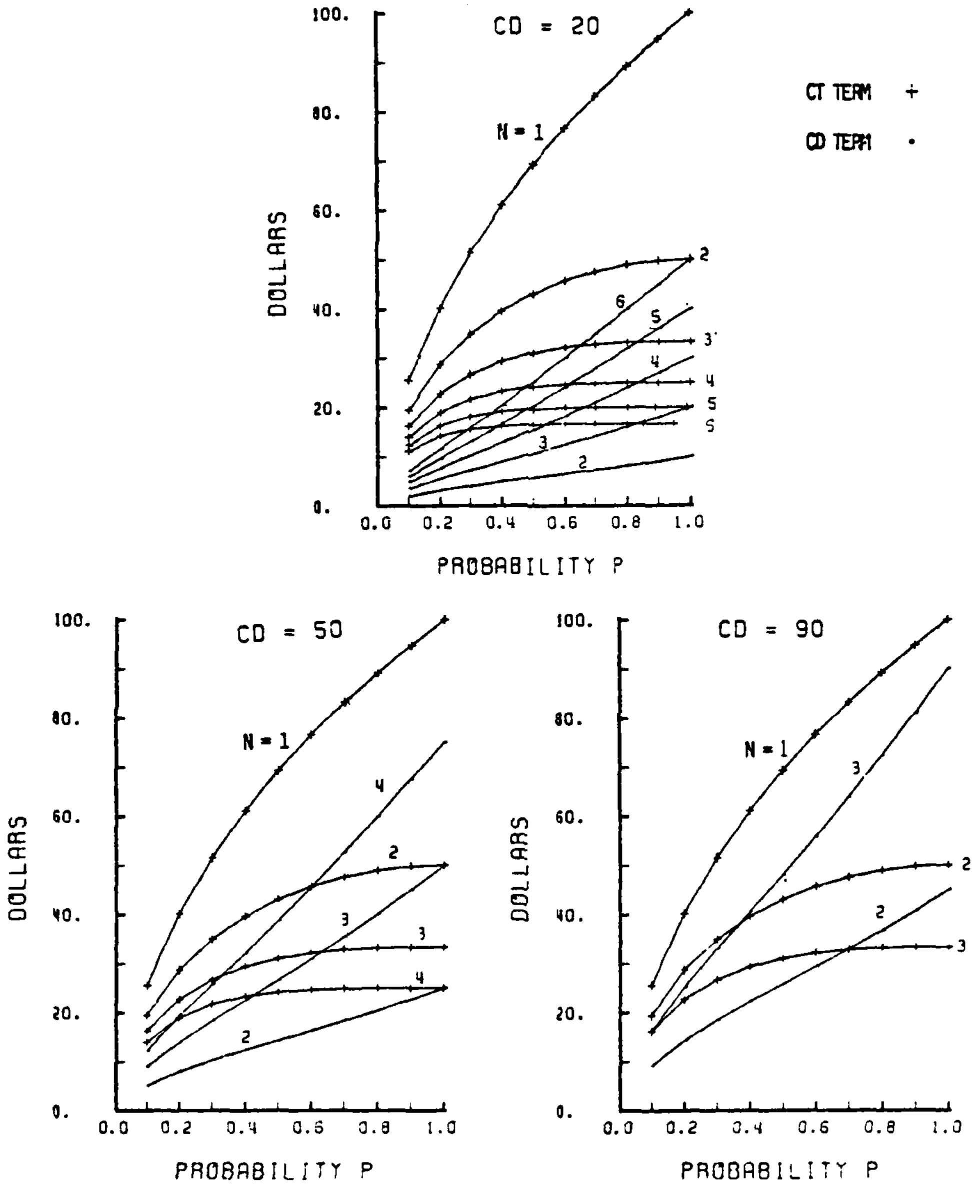

Figure 10. Delivery and Delay Cost Terms for Alternative 1.

Figures 7, 8, and 9 also show that as CD increases, the peaks of the stair-step curves for alternatives 1 and 3 move to the right. By the time CD=90%, the decreasing stair-steps have disappeared and optimal N values have become very small (only 1 and 2). For such small values, the savings in transportation costs in going from N=1 to N=2 remains more than the increase in the delay costs for the higher p values where N=2 is optimal.

The behavior of the cost curves for alternative 1 for various values of N completes the picture. Figures 11, 12, and 13 show the cost curves for several N values for CD = 20%, 50%, and 90%. When CD=50%, note that the curves for N=2 and N=3 cross twice. The N=3 curve produces a more favorable total cost between approximately p=0.2 and p=0.5 while the N=2 curve is less costly for the remaining p values. It is also interesting that for small p values, say less than 0.4, the difference between total cost at optimal N and one greater than or less than the optimal N is not very substantial.

C. THE ALTERNATIVES COMPARED WITH RESPECT TO UNITS DELIVERED AND PERIODS BETWEEN DELIVERIES

The top graph of figure 14 shows the optimum value of N for alternatives 1 and 3 and the expected number of periods between deliveries for alternative 2. As expected, alternative 3 always provides the smallest N values since

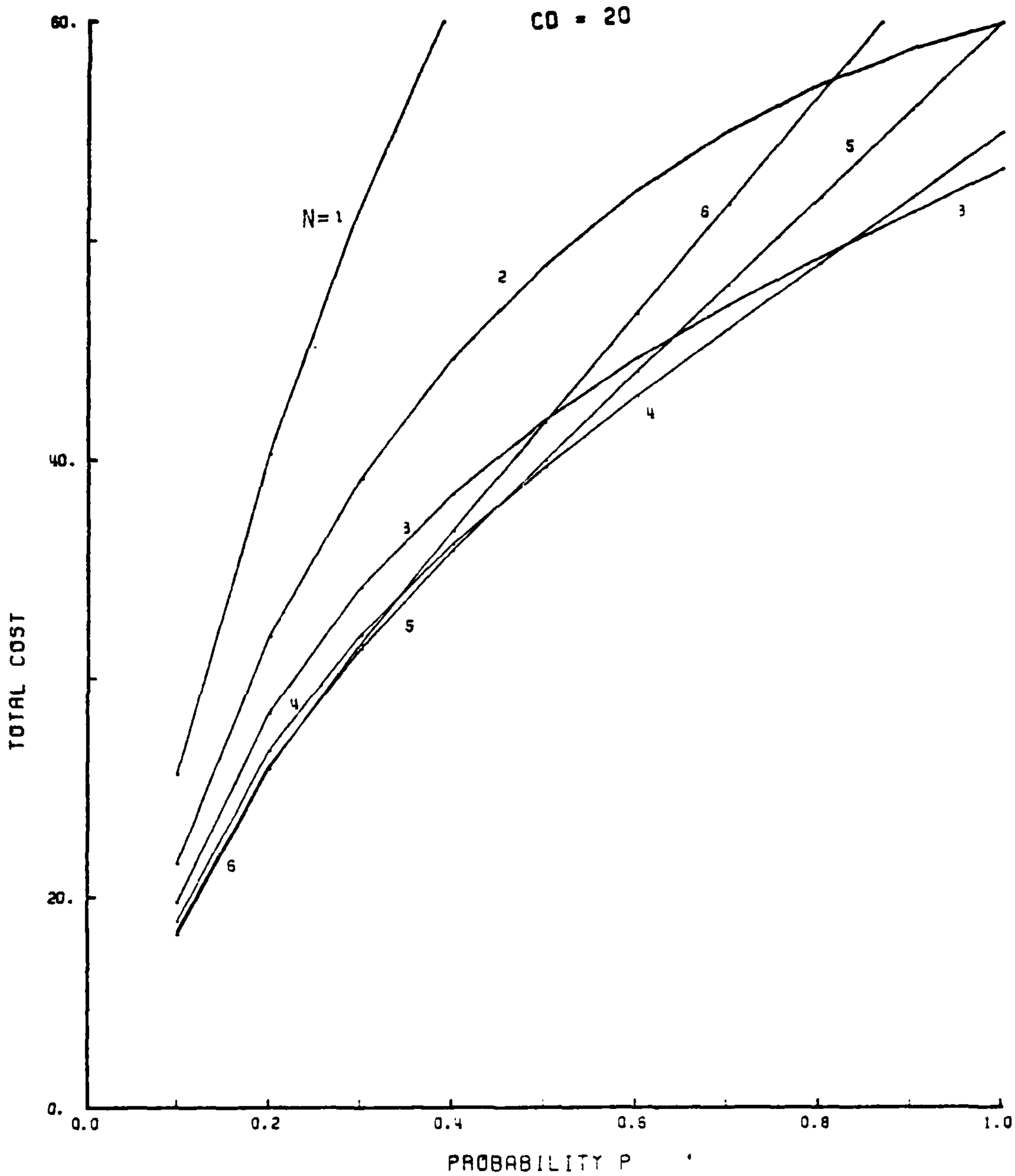

Figure 11. Alternative 1 Total Cost Curves for Specific N Values with CD=20.

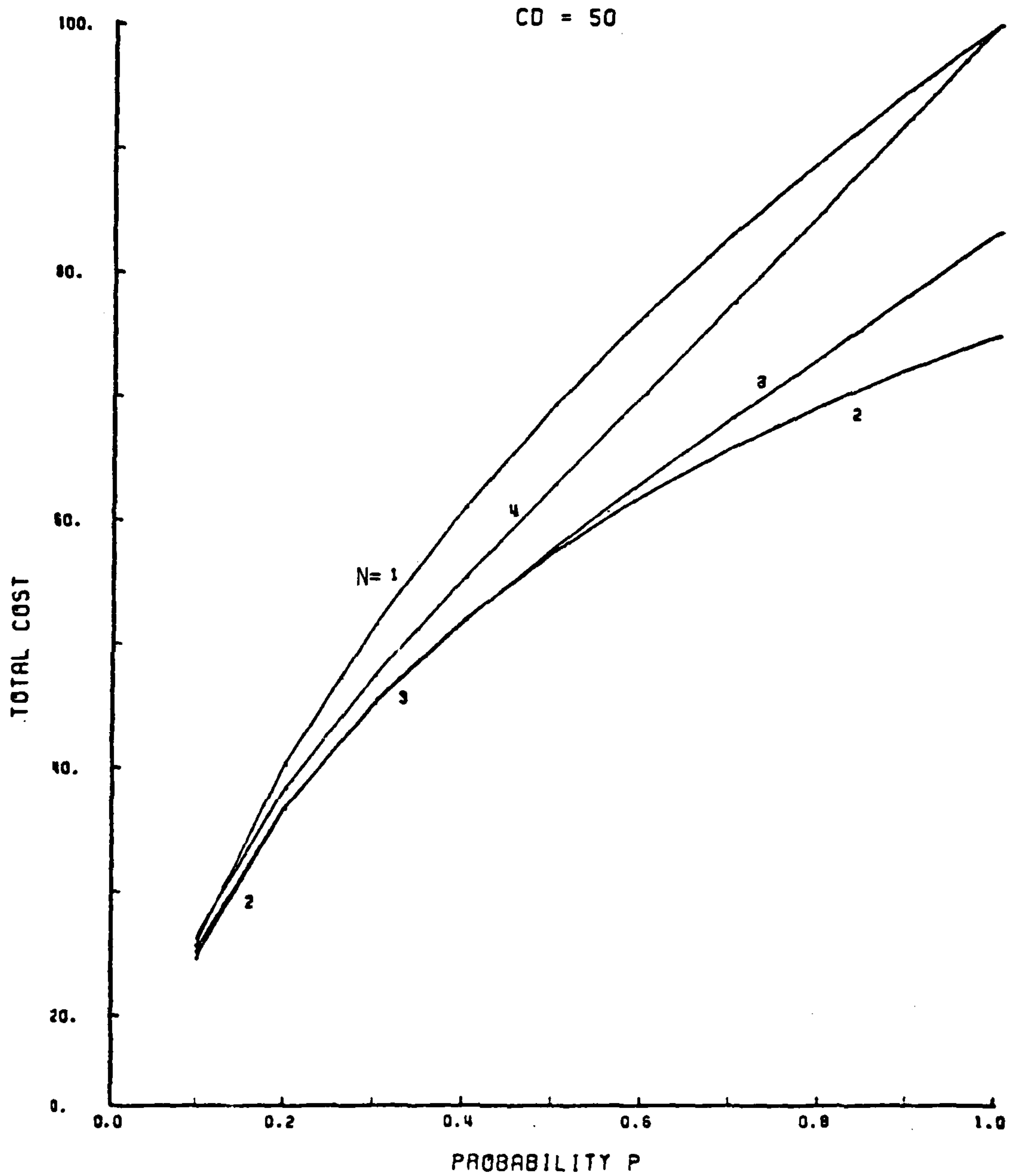

Figure 12. Alternative 1 Total Cost Curves for Specific N Values with CD=50.

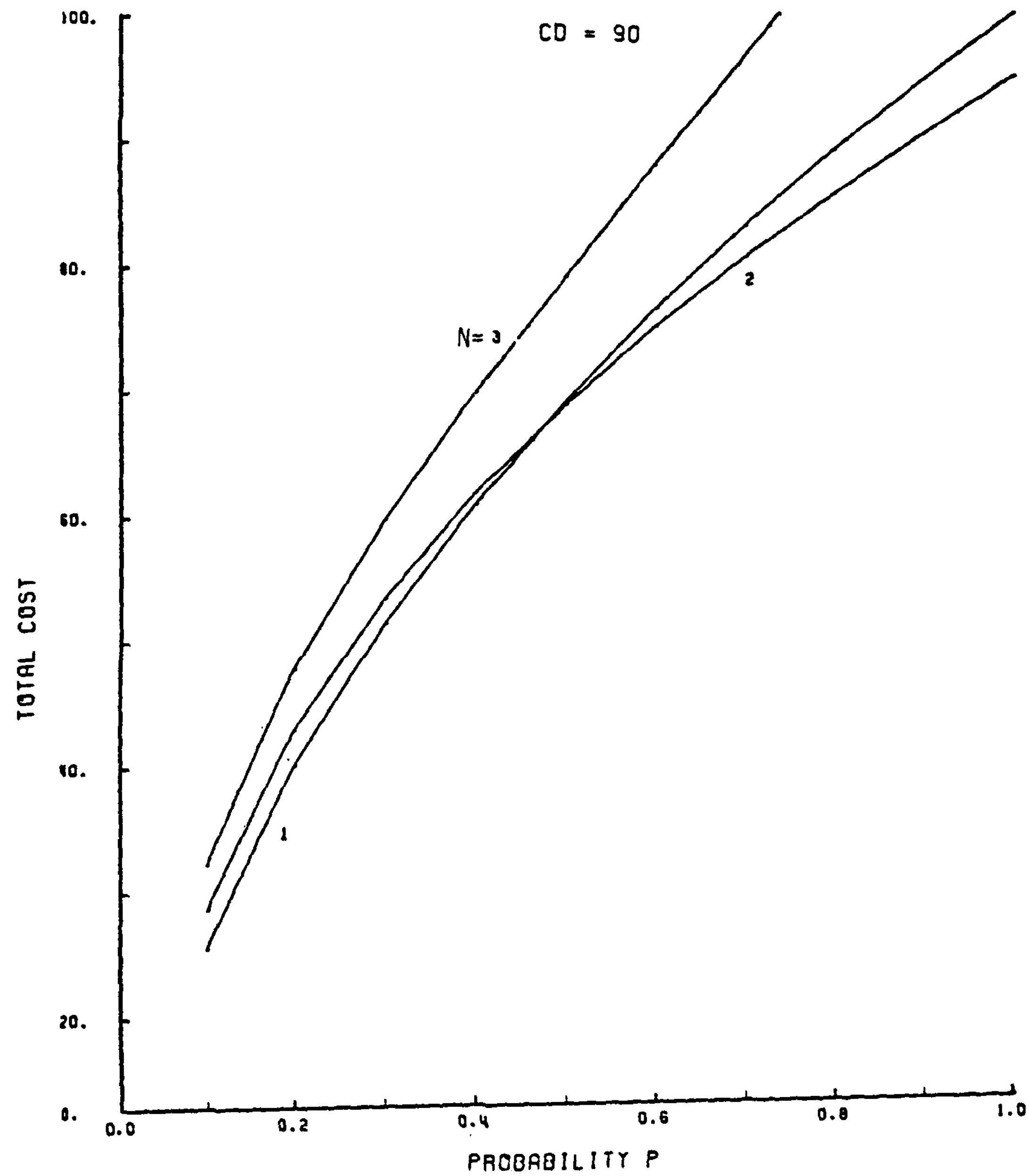

Figure 13. Alternative 1 Total Cost Curves for Specific N Values with CD=90.

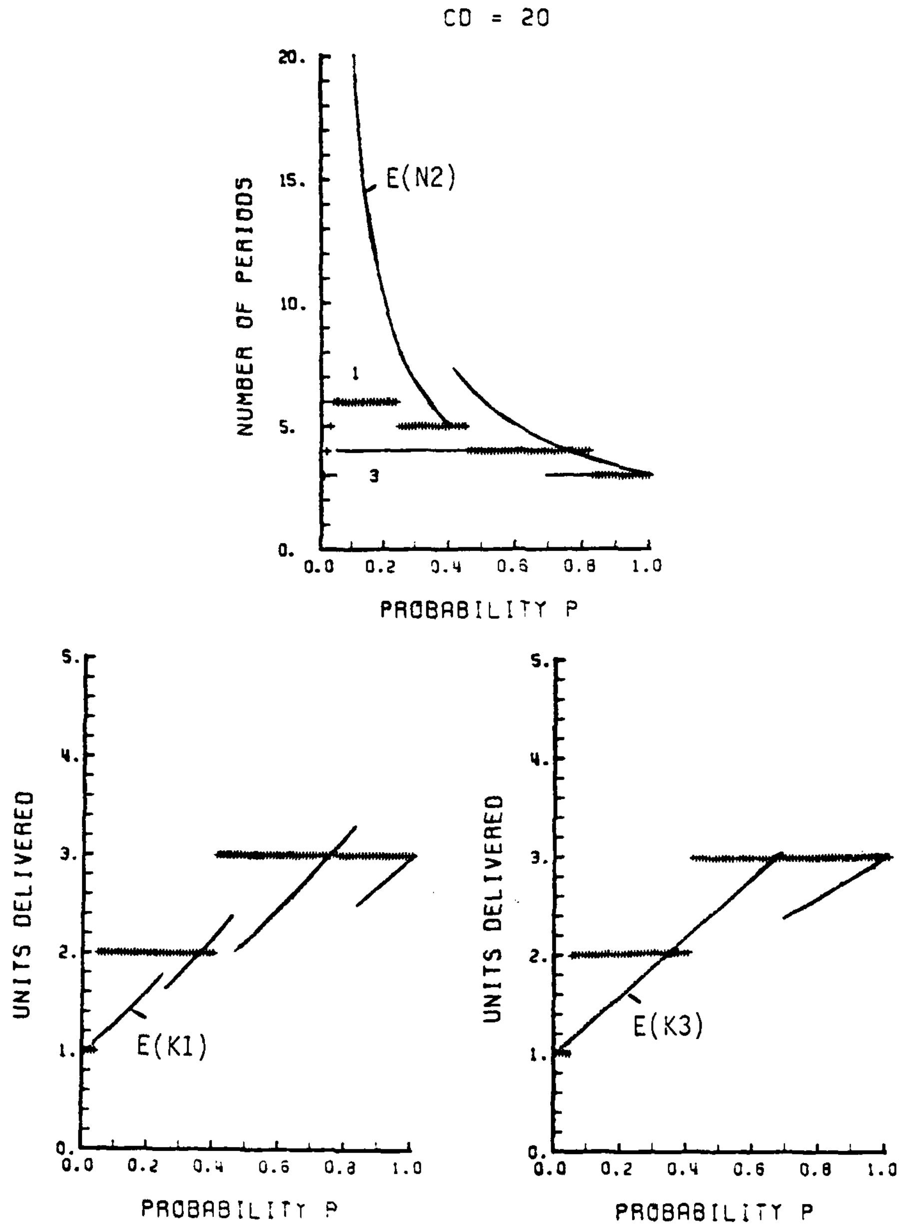

Figure 14. Periods between Deliveries and Units Delivered for All Alternatives.

counting does not begin until the first demand occurs. Except for a narrow range of p values around 0.4, the expected number of periods between deliveries for alternative 2 is larger than the optimum number of periods for alternatives 1 and 3. In addition, the alternative 2 curve goes off to infinity as p goes to zero.

The two bottom graphs of figure 14 compare the optimal number of units delivered under alternative 2 with the expected number of units delivered under alternatives 1 and 3. All alternatives show that the number delivered increases with increasing p even though optimal N for alternatives 1 and 3 increase and then decrease with increasing p. The breaks in the curves for alternatives 1 and 3 correspond to changes in optimal N values.

D. DELAY COST BREAKPOINTS

Figure 15 displays the smallest CD value for which the optimal value for N or K is one versus the probability of a demand. For example, for alternative 1 and a particular p value, it is most economical to schedule deliveries every period for a repair part with a CD value that is greater than or equal to the (p,CD) point on the curve. Suppose that the best estimate of the probability of the demand for a particular repair part is 0.2, under alternative 1 deliveries should be scheduled every period for repair parts with delay costs per period greater than or equal to $72.

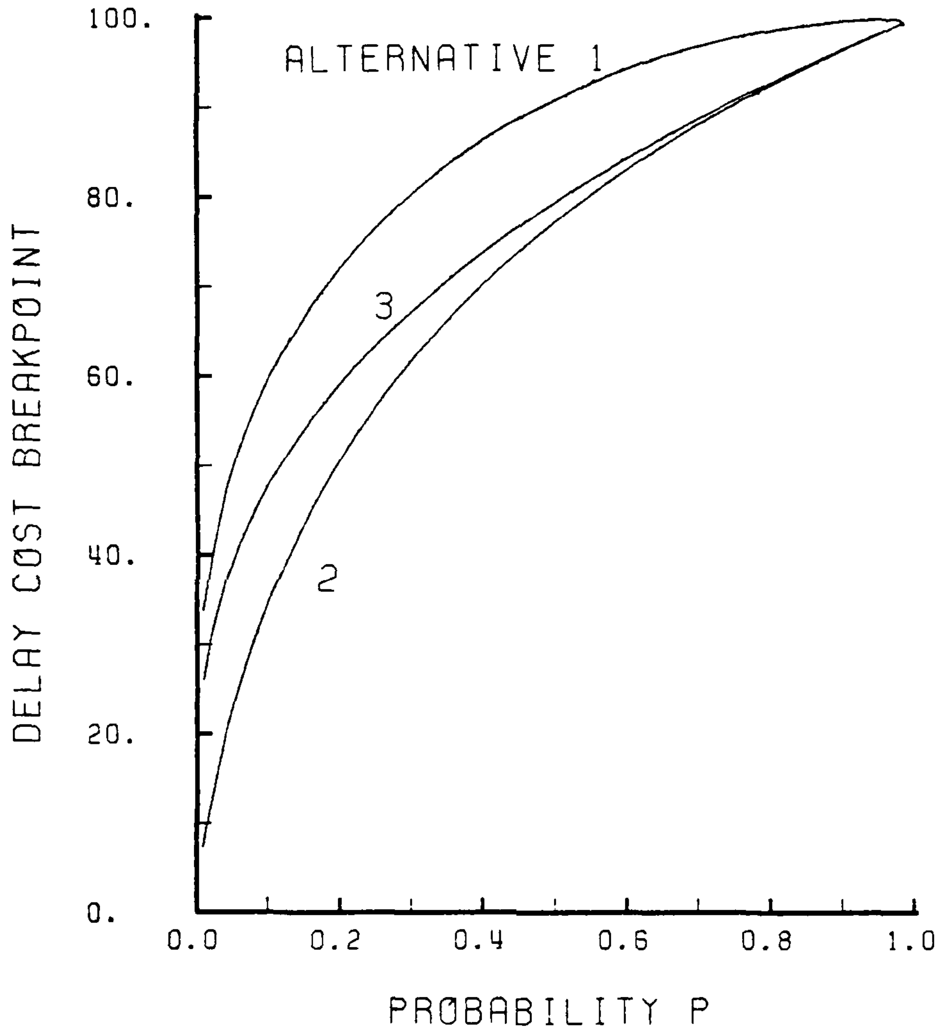

Figure 15. Delay Cost Breakpoints as a Function of p.

This information could be used as a first step in determining whether an item should be held in an on-site inventory at the NARF or as inventory at the NSC. Any repair part with a CD value greater than the applicable point on the curve could be considered for stockage at the NARF. Using this criterion, alternative 1 would result in fewer candidates for stockage at the NARF. At low values of p, alternative 3 would yield fewer candidates than alternative 2, but as p increases the difference between these two curves narrows substantially.

E. THE MODELS UNDER A TIME CONSTRAINT

Time constraints can evolve from several different sources. Two examples are when higher authority dictates system-wide constraints that must be met, or when a time constraint is voluntarily imposed to ensure customer satisfaction. No matter what the source, the random models can be used in the environment of time constraints. The expected delay for each alternative has been derived for just this purpose. In general, if the expected delay for the optimal solution does not exceed the time constraint, the optimal solution remains unchanged. Thus, to be incorporated into the random models, the time constraint must be in the same units as the expected delay, which are the periods for the model. Specifically, if the period being used is 4 days and the time constraint is 2 days, the expected delay may

not exceed [illegible].5 periods or the optimum solution will change. If the constraint alters the optimal solution, the constraint is actually implying a delay cost in excess of that used in the original computations.

IV. A MODIFICATION TO ALTERNATIVE 1

With the implementation of alternative 1, it is reasonable to expect that a delivery truck will be reserved for the scheduled delivery for some time into the future, say for a quarter or even an entire year. However, if no demand occurs up to the time of the scheduled delivery, that delivery would be cancelled. Since this cancellation could not be made until immediately before the delivery was scheduled to have been made, it is also reasonable to expect a charge to be levied against the NSC to cover costs of the reserved but unutilized truck. Currently the PWC does not impose a penalty for cancellation on NSC, Oakland in such circumstances, but it is not unreasonable to expect it in the future, especially if the direct delivery policy increases the number of such cancellations.

Therefore, a modification to alternative 1 to include a penalty for cancellation is desireable. In the following discussion, let PC be the penalty incurred for cancelling one scheduled delivery. For a given N, if there is a demand in the first N periods, the expected cost per period is unchanged from the basic model since no penalty is incurred. If there is no demand in the first N periods and at least one demand in the next N periods, a penalty cost in incurred and the average penalty cost per period is PC/2N. The

associated probability of no demands in the first N periods and at least one demand in the second N periods is

$$(1-p)^N [1 - (1-p)^N] .$$

If there is no demand in the first 2N periods and at least one demand in the third N periods, the average penalty cost per period would be 2PC/3N and the probability of this occurance would be

$$(1-p)^{2N} [1 - (1-p)^N] .$$

In general, the average penalty cost per period for no demands in (k-1)N periods and at least one demand in the last N periods is

$$(k-1)PC/kN. \qquad (11)$$

The associated probability of this occurrence is

$$(1-p)^{(k-1)N} [1 - (1-p)^N] . \qquad (12)$$

Combining equations (11) and (12) and summing over all possible k values yields the expected penalty cost per period as a function of N:

$$EPC(N) = \sum_{k=1}^{\infty} [(k-1)PC/kN] (1-p)^{(k-1)N} [1 - (1-p)^N]$$

$$= PC/N - PC/N \left[1 - (1-p)^N\right] \sum_{k=1}^{\infty} 1/k \, (1-p)^{(k-1)N} .$$

McMasters [Ref.1] has shown the infinite sum is equivalent to

$$[-\ln(1 - (1-p)^N)] / (1-p)^N .$$

Thus, the expected penalty cost per period is

$$PC/N - PC/N\,\left(1-(1-p)^N\right)\left[-\ln\left(1-(1-p)^N\right)/(1-p)^N\right] .$$

This, then, is the additional cost that must be added to equation (2) when a penalty for cancellation of a scheduled delivery is imposed. The total expected average cost now becomes

$$ECP(N) = PC/N + \left[\left(\frac{CT - PC}{N}\right)\left(1-(1-p)^N\right) + pCD(N-1)/2\right]\left[\frac{-\ln\left(1-(1-p)^N\right)}{(1-p)^N}\right] .$$

The concept of a penalty for cancellation does not apply to alternative 2 since a delivery would be scheduled only after N units have been ordered. Neither does it apply to alternative 3, since a delivery would be scheduled (N-1) periods after the first demand was received and the delivery would be made even if there were no more demands up to the delivery time. Although the concept of a cancellation penalty is not reasonable for alternatives 2 and 3, it is reasonable to expect a higher charge for a truck that is not scheduled in advance. If this were the case, the appropriate delivery cost would have to be used in calculations for alternatives 2 and 3 in order for valid comparisons to be made.

Figures 16 and 17 show the affect of the cancellation penalty on the total cost curve for CD=20 and PC = 0, 25, 50, and 100. Careful inspection of figure 16 indicates

slightly higher total costs for larger penalty costs in the low range of p, with very little effect on the total cost as p approaches 1.0. This is reasonable since more delivery cancellations would be expected when the probability of a demand is low. Also as would be expected, the optimum number of periods between deliveries is increased with the penalty, as is seen in figure 17. Finally, the values of optimal k do not increase with p for very low p values as seen in the case where PC>0.

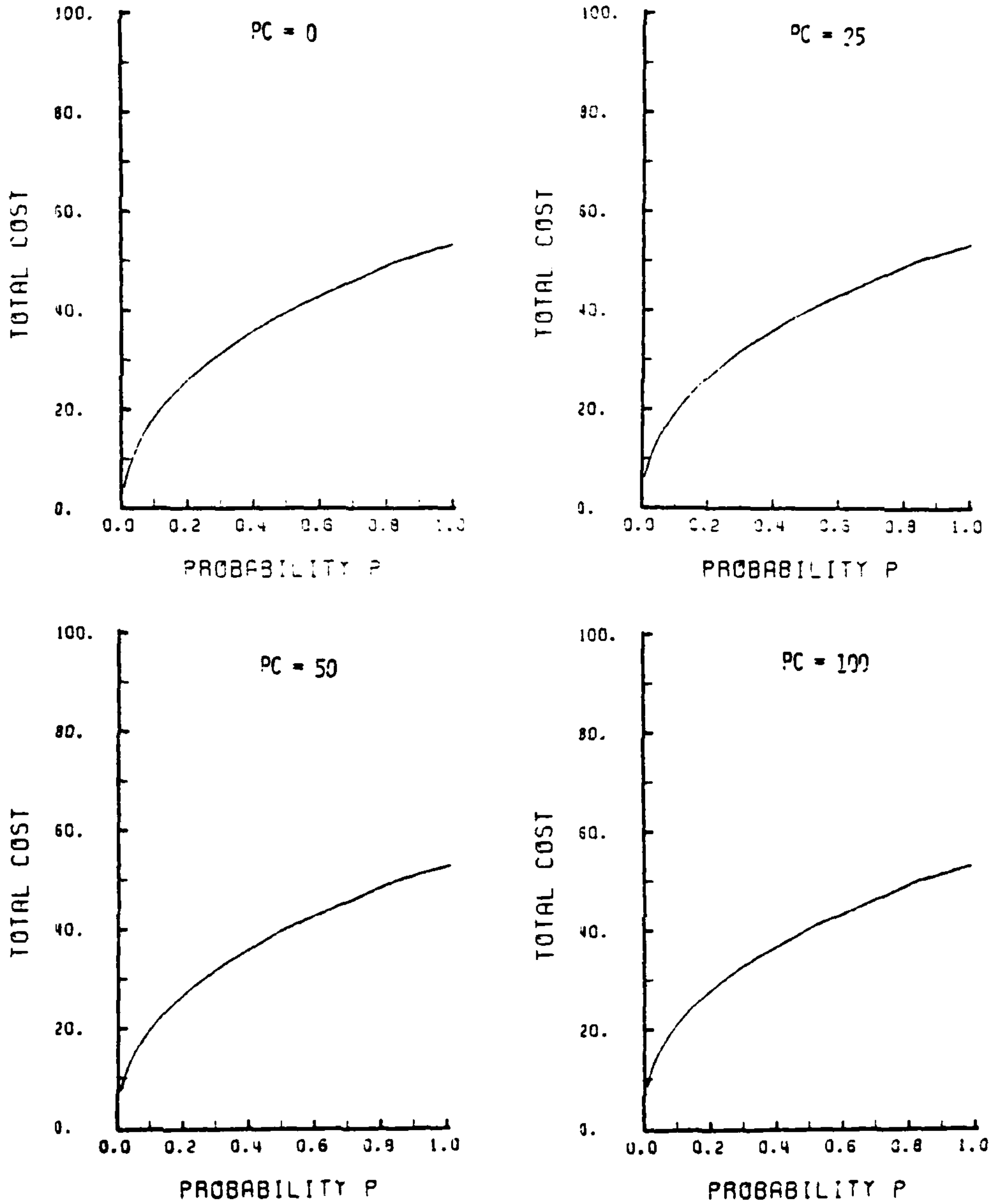

Figure 16. Total Cost Curves for Alternative 1 with Cancellation Penalty and CD=20.

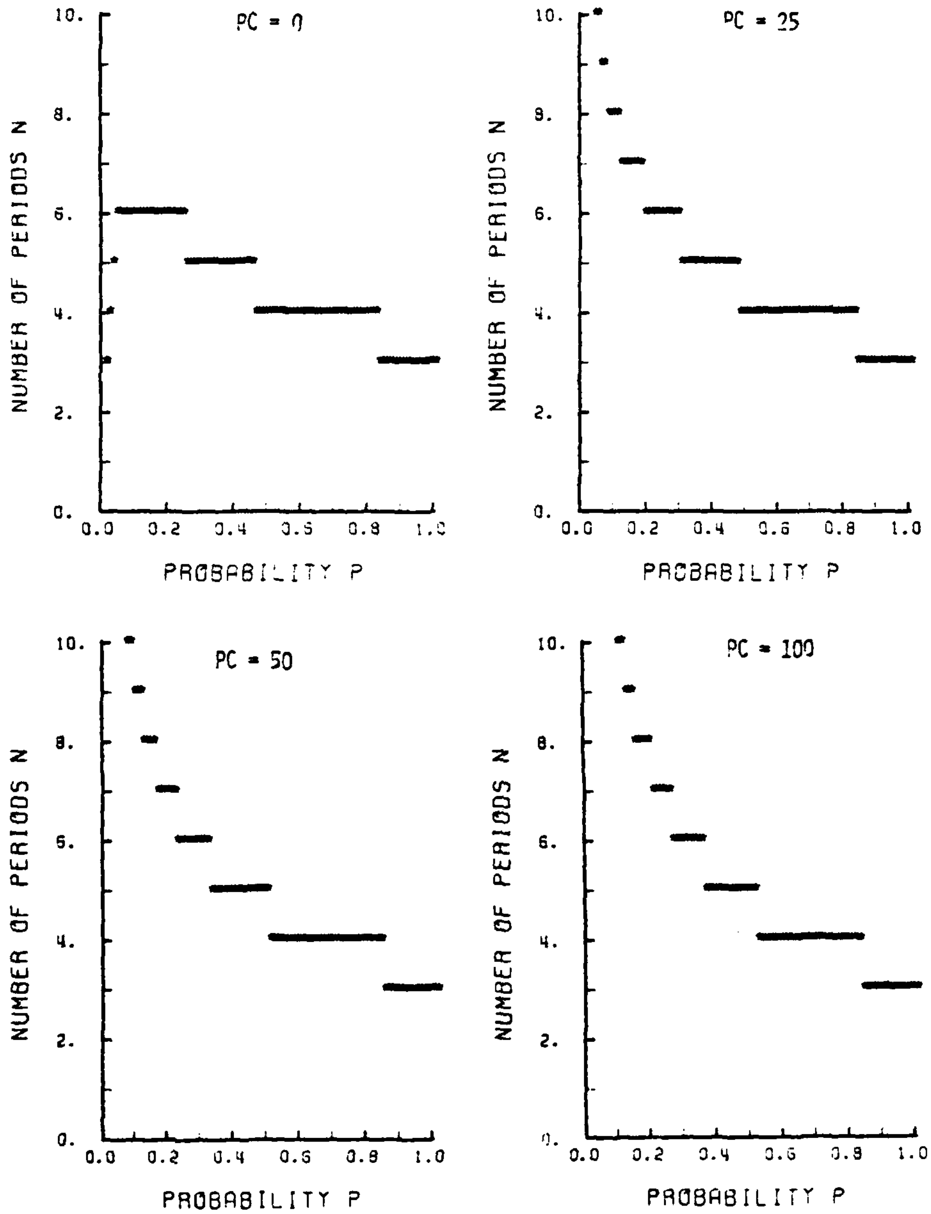

Figure 17. Optimum N for Alternative 1 with Cancellation Penalty and CD=2P.

V. CONCLUSIONS

Probably the most significant and surprising result of the preceding analysis is that there is very little difference in the optimal expected total costs per period among the three alternatives. Although the alternatives differ substantially in form and emphasis, the resulting expected total costs are amazingly close.

The comment was made in Chapter III that the optimal solution is a function of the CD/CT ratio. Thus the parametric analyses also apply to other CT and CD values as long as the ratios are the same as the actual ones used in Chapter III.

When the delivery cost per trip is considered fixed, the parameter with the most impact on the expected total cost is the cost of delay per period (CD). In general, when CD is small, say less than CT/100, the expected total cost is extremely insensitive to changes in p. As CD increases, the expected optimal cost values increase. However, the rate of increase becomes less as p becomes large.

It was also shown that under alternative 1, for small p values, the total cost is rather insensitive to small changes away from optimal N. This is also true for the other two alternatives.

Alternative 3 never produced an optimal cost that was

less than both other alternatives for any p and CD value. It did switch around between second and third best for most of the CD and p value considered and did tie with the others when CD=1 and 12[illegible]. Its expected total cost function was also more complex than those of alternatives 1 and 2. As a consequence, it appears that alternative 3 is not worthy of further consideration.

All things considered, alternative 1 appears to be the most reasonable strategy for an NSC to adopt. It allows trucks to be scheduled in advance, which is by far the least work-intensive alternative. It is often the most cost effective and when it is not, the differences in total cost are small. Even when a penalty for cancellation is incorporated, the total cost changes very little.

If for some reason, implementation of either alternative 2 or 3 is easier, the analysis has shown that the expected total costs will be close. Therefore, the final criterion for which alternative should be chosen is essentially ease of usage and implementation.

LIST OF REFERENCES

1. Naval Postgraduate School Report NPS55-81-011 *Models for Siting Repair Parts Inventories in Support of a Naval Air Rework Facility*, by Alan W. McMasters, April, 1981.

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